

Advanced Project Portfolio Selection Methods

By Jay April, Fred Glover, James P. Kelly, and Manuel Laguna

In all areas of business, determining how to allocate investment capital in order to maximize returns is a ubiquitous challenge where approaches to solutions cover a very wide spectrum. Portfolio optimization for capital investment is often too complex to allow for tractable mathematical formulations. Nonetheless, many analysts force these problems into standard forms that can utilize traditional optimization technologies such as linear and quadratic programming. Unfortunately, such formulations omit key aspects of real world settings resulting in flawed solutions. In this column we focus on a flexible modeling and solution approach that overcomes these limitations.

Background

The beneficiaries of the technology discussed here include executives responsible for capital investments, finance department analysts charged with capital budgeting, and technology managers responsible for project planning and implementation. Their needs provide compelling reasons to use the technology:

- Executives are dissatisfied with their current risk-assessment methods.
- They are under continual pressure to improve capital investment performance.
- They need technology to help communicate the analysis and clearly identify the reasons for specific investment decisions.
- They worry that competitors may adopt new and more advanced technology.

Capital investment decisions are usually accomplished with traditional analyses that include net present value and mean-variance analysis. Consequently most organizations use similar methods to evaluate and select capital spending options.

Many organizations evaluate projects by estimating their net present value (NPV). NPV is calculated by projecting expected future cash flows, “discounting” the future cash flows by the cost of capital, and then subtracting the initial investment. Conventional wisdom directs us to undertake projects if NPV is positive, but this does not guarantee funding. Organizations typically consider other factors, which incorporate their ability to fund the initial investment given their capital structure, current operating cash flow positions, strategic considerations and financial expectations.

In public and private organizations, the decisions of committing limited resources to multiple uses can either strengthen or deteriorate their very financial foundation. On one end of the spectrum, capital budgeting procedures often employ traditional operations research (OR) techniques to guide and support decisions. On the other end, executives admit that selections come down to intuition, combined with seat-of-the-pants “guesstimates,” and peppered with squeaky wheel assignments. Typically, however, what is common is to build models that employ pro-forma plans centering on measures of the benefits of the investments – returns, payback period, and risk. The list may expand to include cash flow, cost of capital, market share, etc.

Evaluations of alternatives are also made in a variety of ways, from one-at-a-time comparisons of returns and risks to more sophisticated portfolio optimization and real options. In companies using these sophisticated methods, portfolio selection usually includes mean-variance analysis.

In a seminal paper in 1952 in the *Journal of Finance*, Nobel laureate Harry Markowitz laid down the basis for modern portfolio theory. Markowitz focused the investment profession’s attention to *mean-variance efficient portfolios*. A portfolio is defined as mean-variance efficient if it has the highest expected return for a given variance, or if it has the smallest variance for a given expected return.

In practice, mean-variance efficient portfolios have been found to be quite unstable. Typically, input parameters like expected returns, correlation among projects and project variance are estimated using either historical data or forecasts. Researchers have found that estimation errors in the input parameters overwhelm the theoretical benefits of the mean-variance paradigm.

Although cracks in the foundation are becoming too conspicuous to ignore, capital budgeting participants have been dedicated to traditional ideas for so long that they are not able to pull away, even at the expense of severely hampering their financial growth. More progressive analysts have insistently sounded the alert about the crumbling structure underlying mainstream investment strategies. Still, the best response has been to cobble together various ad-hoc measures in an attempt to shore up the framework, or erect a makeshift alternative. Recognition that this response is far from ideal has persuaded many to cling to the old ways, in spite of their apparent defects.

The inability to devise a more effective alternative has been due in large part to limitations in the technology of decision-making and analysis, which has offered no reliable method to conquer the complexity of problems attended by uncertainty. As a result, the goal of evaluating investments effectively, and accurately account for tradeoffs between risk and potential return, has remained incompletely realized and ripe for innovation.

Over the last several years, alternative methods have emerged for optimizing decisions under uncertainty. New technologies have begun to penetrate planning and decision-making in business, making it possible to develop solutions to more realistic models. In capital budgeting and investment, agent-based modeling, genetic algorithms, and real options analysis all seek to improve on traditional methods by introducing more flexible, robust, and realistic assumptions and providing more sophisticated analysis and forecasting tools.

Optimization Method

Due to the complexity and uncertainty in real systems, simulation often becomes a basis for handling complex decisions. Advances in the field of metaheuristics – the domain of optimization that augments traditional mathematics with artificial intelligence

and methods based on analogs to physical, biological, or evolutionary processes – have led to the creation of optimization engines that successfully guide a series of complex evaluations (simulations) with the goal of finding optimal values for decision variables. One example is the search algorithm embedded in the OptQuest[®] optimization system developed by OptTek System, Inc. OptQuest[®] is designed to search for optimal solutions to the following class of optimization problems:

Max or Min	$F(x)$	(Objective)
Subject to		
$Ax \leq b$	(Constraints)	
$g_l \leq G(x) \leq g_u$	(Requirements)	
$l \leq x \leq u$	(Bounds)	

where x can be continuous or discrete. The objective function, $F(x)$, may be any mapping from a set of values x to a real value. The set of constraints must be linear and the coefficient matrix “A” and the right-hand-side values “b” must be known. The requirements are simple upper and/or lower bounds imposed on a linear or non-linear function. The values of the bounds “ g_l ” and “ g_u ” must be known constants. All variables must be bounded and some may be restricted to be discrete with an arbitrary step size.

A typical example might be to maximize the NPV of a portfolio by judiciously choosing projects subject to budget restrictions and a limit on risk. In this case, x represents specific project participation levels, and $F(x)$ is the expected NPV. The budget restriction is modeled as $Ax \leq b$ and the limit on risk is a requirement modeled as $G(x) \leq g_u$ where $G(x)$ is a percentile value. Each evaluation of $F(x)$ and $G(x)$ requires a Monte Carlo simulation of the portfolio. By combining simulation and optimization, a powerful tool results.

The optimization procedure uses the outputs from the system evaluator (simulation), which measures the merit of the inputs that were fed into the model. On the basis of both current and past evaluations, the optimization procedure decides upon a new set of input values (see Figure 1).

The optimization procedure is designed to carry out a special search, where successively generated inputs produce varying evaluations, which over time provide a highly efficient trajectory to the best solutions. The process continues until an appropriate termination criterion is satisfied (usually based on users’ preference for the amount of time to be devoted to the search).

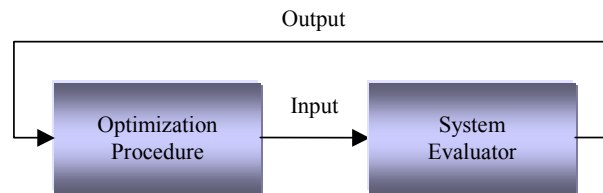


Figure 1- Coordination Between Optimization and Evaluation

OptQuest[®] seeks to find an optimal solution to a problem defined on a vector x of bounded variables. The user can define several types of optimization problems depending on the combination of variables:

- Pure continuous
- Pure discrete (including pure binary problems)
- Mixed problems (continuous-discrete, continuous-permutation, discrete-permutation or continuous-discrete-permutation)

The optimization problem may be unconstrained, or include linear constraints and/or requirements. The optimizer detects certain problem structures to trigger a complete enumeration routine that guarantees optimality.

The optimization technology used here relies primarily on the metaheuristic known as *scatter search* (OptQuest actually employs a combination of scatter search, advanced tabu search, linear and mixed integer programming, neural networks, and linear regression). Scatter search is designed to operate on a set of points, called *reference points*, which constitute good solutions obtained from previous efforts. The basis for defining “good” includes special criteria – such as diversity – that go beyond the objective function value. The approach systematically generates new points from combinations of the reference points. The combinations are generalized forms of linear combinations, accompanied by processes to adaptively enforce constraint-feasibility and encourage requirement-feasibility.

Points are considered diverse if their elements are “significantly” different from one another. The optimizer uses Euclidean distances to determine how “close” a potential new point is from those in the reference set, in order to decide whether the point is included or discarded. The number of new solutions created depends on the quality of the solutions being combined. Specifically, when the best two reference solutions are combined, they generate up to 5 new solutions, while when the worst two are combined they generate only one.

In the process of searching for a global optimum, the combination method may not be able to generate solutions of enough quality to become members of the reference set. If the reference set does not change and all the combinations of solutions have been explored, a diversification step is triggered. This step consists of rebuilding the reference set to create a balance between solution quality and diversity. To preserve quality, a small set of the best (*elite*) solutions in the current reference set is used to seed the new reference set. Then, the diversification method is used to repopulate the reference set with solutions that are diverse with respect to the elite set. This reference set is used as the starting point for a new round of combinations. This method guarantees that a very good solution is found quickly.

The integration of simulation with optimization has been shown to be a powerful approach for portfolio optimization. At times, however, the computational cost of multiple simulations can be quite high. At the beginning of 2003, OptTek received a National Science Foundation Small Business Innovative Research (SBIR) award to develop a method to minimize the number of simulations required to determine the optimal project portfolio. The method, which is called the layer envelope response method, is forecast to significantly improve the efficiency in achieving the optimal solutions.

Project Portfolio Optimization

In industry, strategic planning requires the selection of a portfolio of projects to advance the corporate goals. In general, there are more projects than funding can support so the selection process must choose a subset of projects to maximize the company's profit goals while obeying budgetary restrictions. Concurrently, executives wish to manage the overall risk of the portfolio while ensuring that cash flow and other such requirements are satisfied.

An Illustrated Example

The Petroleum and Energy (P&E) industry uses project portfolio optimization to manage investments in oil and gas exploration and production. Each project's proforma is modeled as a simulation capturing the uncertainties of production and sales.

The following example involves ten-year models of five potential projects with multiple types of uncertainty in drilling, production, and market conditions. For our analysis, we used Crystal Ball[®], by Decisioneering, to simulate the set of projects, and OptQuest[®], by OptTek Systems, Inc. to guide the selection of the best portfolio. We examined multiple cases to demonstrate the flexibility of the optimization engine to enable a variety of decision alternatives.

Case 1: Traditional Markowitz Approach

In case 1, the decision was to determine participation levels [0,1] in each project with the objective of maximizing the expected NPV of the portfolio while keeping the standard deviation of the NPV below a specified threshold. In this case, all projects must begin in the first year.

Objective: Maximize $E(\text{NPV})$

Requirement: Keep $\sigma \leq \$10,000\text{M}$

Constraints: All projects must start in year 1

The optimal portfolio resulted in an expected NPV of \$37,400M with a standard deviation of \$9,500M. Figure 2 shows the corresponding non-normal NPV distribution.

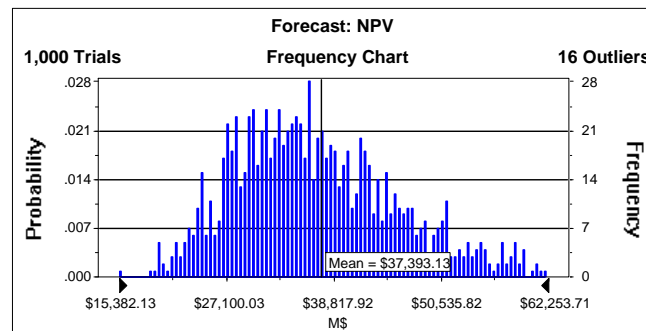


Figure 2 - Case 1 NPV Distribution

Case 2: Varying Starting Times

In case 2, the goal was to determine participation levels in each project where starting times could vary. We maximized the probability of exceeding the expected NPV of \$47,500M (achieved in Case 1). Limiting the 10th percentile of NPV controlled risk.

Objective: Maximize $\text{Probability}(E(\text{NPV}) \geq \$47,455\text{M})$

Requirement: Keep 10th Percentile of NPV $\geq \$36,096\text{M}$

Constraints: All projects may start in year 1, year 2, or year 3

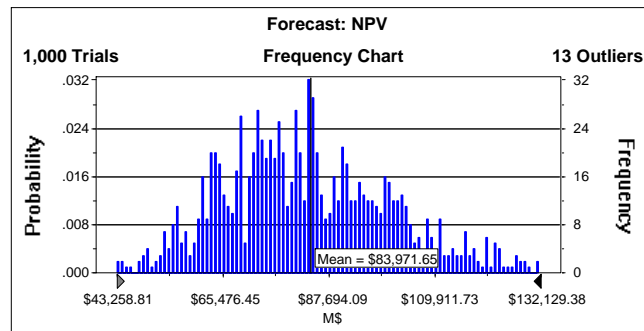


Fig. 3- Case 2 NPV Distribution

In this case, the best portfolio resulted in an expected NPV of approximately \$84,000M with a standard deviation of \$18,500M. NPV has a 99% probability of exceeding \$47,500M (Figure 3). This case demonstrates that adopting measures of risk other than standard deviation can result in superior portfolios. Simulation optimization is the only technology that can offer these types of analyses.

Conclusion

The approach discussed here brings intelligence to software for corporate decision-making, and gives a new dimension to optimization and simulation models in business and industry. The method empowers decision makers to look beyond conventional decision-making approaches and actually pinpoint the most effective choices in uncertain situations. The implementation of the software should allow senior management to confidently maximize financial return while accurately measuring and controlling risk.

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